A comparative analysis of tunneling time concepts: Where do transmitted particles start from, on the average?

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(Dated: February 1, 2008)

In this paper we compare the concept of the tunneling time introduced in quant-ph/0405028 with those of the phase and dwell times. As is shown, unlike the latter our definition of the transmission time coincides, in the limit of weak scattering potentials, with that for a free particle. This is valid for all values of the particle's momentum, including the case of however slow particles. All three times are also considered for a resonant tunneling. In all the cases the main feature to distinguish our concept from others is that the average starting point of transmitted (reflected) particles does not coincide with that of all particles. One has to stress here that there is no such an experiment which would give coordinates of all the three points, simultaneously. For measuring the position of the average starting point of transmitted particles we propose an experimental scheme based on the Larmor precession effect.

I. INTRODUCTION

As is known [1, 2, 3, 4, 5, 6], the question of the time spent by a quantum particle in the barrier region is still controversial. At present there are many different definitions of this quantity, however none of them is commonly accepted. At the same time, for a given potential and initial state of a particle, i.e., in the standard setting of the tunneling problem, the above question is evident to imply an unique answer.

In our previous paper [7] we have introduced the concepts of the transmission and reflection times based on a separate description of the transmission and reflection processes. By our approach, tunneling is a combined random process to consist from two alternative elementary ones, transmission and reflection. For a given potential and initial state of a particle, we have found an unique pair of solutions to the Schrödinger equation, which describe separately these processes. This permits one to follow the centers of "mass" (CMs) of the transmitted and reflected wave packets, at all stages of scattering, and, as a consequence, to calculate the times spent by the CMs in the barrier region. These characteristic times are treated in our formalism as the (average) transmission and reflection times for a particle.

Note, unlike the standard wave-packet analysis (SWPA) our approach predicts that the average starting point of transmitted (and reflected) particles does not coincide with the starting point of the CM of the incident wave packet to describe all particles. By this reason, our transmission and reflection times differ essentially from the corresponding phase times derived in the SWPA.

We have to stress that our approach agrees entirely with the foundations of quantum theory, for the wave functions for transmission and reflection to underlay it are solutions to the Schrödinger equation. Besides, one has to bear in mind that the average starting point of transmitted particles and that of all scattering particles cannot be measured simultaneously, in principle.

In this paper we show that the formal comparison of the well-known phase and dwell times with ours speaks in favor of the latter. And, what is more important, our formalism can be experimentally verified. Due to the Larmor precession of the particle's spin in an external magnetic field applied beyond the barrier region, one can measure, in principle, the average starting point of transmitted (or reflected) particles.

Note, apart from the phase time, the dwell and Larmor time are, perhaps, the most cited concepts of the tunneling time. Thus, with taking into account that the Larmor time coincides with the dwell time, it is useful to compare our concept of the tunneling time with those of the phase and dwell times.

II. CRITIQUE OF THE DWELL-TIME CONCEPT

In [7] we pointed to the principal shortcomings of the phase time concept. Now, before a formal comparing of the three concepts, we want to dwell on the principal shortcomings of the dwell time concept.

We begin with the fact that calculating the average values of the particle's position and momentum makes sense only when this calculation is performed separately for the subensembles of transmitted and reflected particles. As regards the average values of these quantities calculated over the whole ensemble of particles, they behave non-causally in the course of the scattering process and hence cannot be interpreted as the expectation values of the position and momentum of a scattering particle.

By our approach, introducing an observable to describe tunneling, without distinguishing transmission and re-

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flection, is meaningless. Quantum mechanics does not imply computing the expectation values of physical quantities for a tunneling particle, which would be common for transmission and reflection. Figuratively speaking, it is merely impossible to "pack up" the properties of the alternative processes into one common characteristics. This concerns entirely the dwell time.

Thus, by our approach, introducing this time scale is questionable from the most beginning. Besides, it is evident that a proper definition of the transmission time for any spatial interval should be valid irrespective of the displacement of this interval on the OX-axis. However, the definition of the dwell time is evident to violate this requirement. Indeed, as is known, this time is defined as the ratio of the probability to find a particle within the barrier region to the incident flux. We consider this definition as a purely speculative one. For the used here normalization by the incident flux has no solid physical basis.

Let us assume, for example, that we study the motion of a particle in the spatial region of the same width but shifted, with respect to the barrier region, toward the transmission domain. What flux should be used in defining the dwell time for this region? Of course, if this spatial region lies entirely in the transmission region, then it is naturally to use for this purpose the transmitted flux (and, as a consequence, a resulting dwell time will describe transmitted particles only). However, if the shifted spatial region coincides partly with the barrier region, then neither incident nor transmitted flux cannot be used in the above definition.

Consider another example. Let the spatial region investigated be the right half of the localization region of the rectangular potential barrier. At the first glance, the dwell time for this case should be the ratio of the probability to find a particle within this region to the incident flux. However, by our approach, if the particle impinges the barrier from the left, then in the case of reflection it never enters the right half of a symmetrical potential barrier. It is evident to be meaningless to take into account the reflected part of the quantum ensemble of particles, in timing a particle in this spatial region.

III. RECTANGULAR POTENTIAL BARRIERS

Now we proceed to a formal comparison of the properties of our time scale with those of the phase and dwell times. Note firstly that in any approach the tunneling time for the particle with a given value of k can be written as $mD_{eff}(k)/\hbar k$. In this paper we analyze the behaviour of $D_{eff}(k)$, in the case of rectangular potential barriers, for the phase (τ_{phase}), dwell (τ_{dwell}) and tunneling time (τ_{tun}) introduced in our previous paper [7]. Of course, unlike τ_{phase} and τ_{tun}, τ_{dwell} describes all particle of a quantum ensemble. However, one has to bear in mind that in the case of rectangular potential barriers the tunneling times for transmission and reflection are equal. It

is naturally to expect that in this case the dwell time should coincide by value with the transmission time.

Taking into account the expression for τ_{phase}, τ_{dwell} (see, for example, [8]) and τ_{tun}, one can easily to obtain the expressions for the corresponding effective barrier's widths. Let

$$\tau_{phase}(k) \equiv \frac{m}{\hbar k}D_{phase}(k), \quad \tau_{dwell}(k) \equiv \frac{m}{\hbar k}D_{dwell}(k),$$

and

$$\tau_{tun}(k) \equiv \frac{m}{\hbar k}d_{eff}(k).$$

For the below-barrier case ($E \leq V_0$) we have

$$D_{phase} = \frac{1}{\kappa} \cdot \frac{2\kappa dk^2(\kappa^2 - k^2) + \kappa_0^4 \sinh(2\kappa d)}{4k^2\kappa^2 + \kappa_0^4 \sinh^2(\kappa d)},$$

$$D_{dwell} = \frac{k^2}{\kappa} \cdot \frac{2\kappa d(\kappa^2 - k^2) + \kappa_0^2 \sinh(2\kappa d)}{4k^2\kappa^2 + \kappa_0^4 \sinh^2(\kappa d)},$$

$$d_{eff} = \frac{4}{\kappa} \cdot \frac{\left[k^2 + \kappa_0^2 \sinh^2(\kappa d/2)\right]\left[\kappa_0^2 \sinh(\kappa d) - k^2\kappa d\right]}{4k^2\kappa^2 + \kappa_0^4 \sinh^2(\kappa d)}$$

where $\kappa = \sqrt{2m(V_0 - E)/\hbar^2}$.

For the above-barrier case ($E \geq V_0$) —

$$D_{phase} = \frac{1}{\kappa} \cdot \frac{2\kappa dk^2(\kappa^2 + k^2) - \kappa_0^4 \sin(2\kappa d)}{4k^2\kappa^2 + \kappa_0^4 \sin^2(\kappa d)},$$

$$D_{dwell} = \frac{k^2}{\kappa} \cdot \frac{2\kappa d(\kappa^2 + k^2) - \kappa_0^2 \sin(2\kappa d)}{4k^2\kappa^2 + \kappa_0^4 \sin^2(\kappa d)},$$

$$d_{eff} = \frac{4}{\kappa} \cdot \frac{\left[k^2 - \beta\kappa_0^2 \sin^2(\kappa d/2)\right]\left[k^2\kappa d - \beta\kappa_0^2 \sin(\kappa d)\right]}{4k^2\kappa^2 + \kappa_0^4 \sin^2(\kappa d)}$$

where $\kappa = \sqrt{2m(E - V_0)/\hbar^2}$; $\beta = 1$ if $V_0 > 0$, otherwise, $\beta = -1$. In both the cases $\kappa_0 = \sqrt{2m|V_0|/\hbar^2}$.

In [7] we treated the case when the potential barrier is localized in the region $[a, b]$, and the CM of the incident wave packet to describe all particles is, at $t = 0$, at the point $x = 0$; $a > 0$. Remind that in accordance with the standard wave-packet analysis namely this spatial point should be considered as the average starting point both for transmitted and for reflected particles. However, by our approach, this is not the case. As was shown in [7], the average starting points, x_{start}^{tr} and x_{start}^{ref}, of transmitted and reflected particles, respectively, do not coincide with that for all particles.

For the above initial condition, in the case of symmetrical potential barriers, we have $x_{start}^{tr}(k) = x_{start}^{ref}(k) = x_{start}(k)$ where

$$x_{start}(k) = -2\frac{\kappa_0^2}{\kappa} \frac{(\kappa^2 - k^2)\sinh(\kappa d) + k^2\kappa d\cosh(\kappa d)}{4k^2\kappa^2 + \kappa_0^4 \sinh^2(\kappa d)},$$

$$x_{start}(k) = -2\beta\frac{\kappa_0^2}{\kappa} \cdot \frac{(\kappa^2 + k^2)\sin(\kappa d) - k^2\kappa d\cos(\kappa d)}{4k^2\kappa^2 + \kappa_0^4\sin^2(\kappa d)},$$

for $E < V_0$ and $E \geq V_0$, respectively. Note, these quantities are such that (see [7])

$$D_{phase}(k) = d_{eff}(k) - x_{start}(k), \qquad (1)$$

from which a formal connection between the phase-time concept and ours is seen explicitly.

IV. CHARACTERISTIC TIMES IN THE LIMIT OF WEAK SCATTERING POTENTIALS

As is known, by the standard in quantum mechanics timing procedure (based on timing the CM of a wave packet), for a free particle with the well defined momentum $\hbar k$ the average time spent by the particle in the spatial region of width d is equal to $\tau_{free}(k; d)$ where $\tau_{free}(k; d) = md/\hbar k$; m is the particle's mass, $k = \sqrt{2mE/\hbar^2}$, E is the particle's energy. This expression is valid for any value of k. In particular, in the limit $k \to 0$, $\tau_{free}(k; d)$ diverges as k^{-1}.

It is obvious that when the potential energy of a particle in the barrier region diminishes (this case is named here as the limit of weak scattering potentials) then $D_{eff}(k)$ should approach d, for any value of k. In the limit of weak scattering potentials the k-dependence of a true tunneling time must approach the function $\tau_{free}(k; d)$. In particular, for infinitesimal potentials the transmission time should diverge as k^{-1} when $k \to 0$. This requirement should be considered as a touchstone in solving the tunneling time problem. For example, in the case of the rectangular barrier of width d and height V_0 this should take place when $V_0 d \to 0$.

At the first glance, all three times behave properly in the limit of weak scattering potentials. Indeed, in all the cases, if $k \neq 0$, $D_{eff}(k)/d \to 1$ when $\kappa_0 d \to 0$. But, as it was stressed above, this should be valid also for however small values of k. As will be seen from the following, only τ_{tun} obeys this requirement.

One can easily show that for $\kappa_0 d \neq 0$ and $k = 0$

$$\frac{D_{phase}}{d} = \frac{2}{\kappa_0 d\tanh(\kappa_0 d)}, \qquad \frac{D_{dwell}}{d} = 0, \qquad (2)$$

$$\frac{d_{eff}}{d} = \frac{2}{\kappa_0 d}\tanh\left(\frac{\kappa_0 d}{2}\right), \frac{x_{start}}{d} = -\frac{2}{\kappa_0 d\sinh(\kappa_0 d)} \quad (3)$$

for $V_0 > 0$; for $V_0 < 0$

$$\frac{D_{phase}}{d} = -\frac{2}{\kappa_0 d\tan(\kappa_0 d)}, \qquad \frac{D_{dwell}}{d} = 0, \qquad (4)$$

$$\frac{d_{eff}}{d} = \frac{2}{\kappa_0 d}\tan\left(\frac{\kappa_0 d}{2}\right), \qquad \frac{x_{start}}{d} = \frac{2}{\kappa_0 d\sin(\kappa_0 d)}. \quad (5)$$

Note, in the long-wave limit, $D_{dwell}/d = 0$ irrespective of $\kappa_0 d$. The corresponding limits for the phase time and τ_{tun} depend on $\kappa_0 d$. Moreover, for $k = 0$, as $\kappa_0 d \to 0$, $D_{phase}/d \to \infty$ for barriers ($V_0 > 0$), and $D_{phase}/d \to -\infty$ for wells ($V_0 < 0$). As regards our definition, in this limit $d_{eff}/d = 1$ both for barriers and wells.

Figs. 1-4 show the dependence of D_{eff}/d on E/V_0, for all three characteristic times, for weak scattering potentials. Figs. 1, 2 correspond to the narrow ($d = 0.5nm$) rectangular barrier ($V_0 = 0.25eV$) and well ($V_0 = -0.25eV$), respectively. Figs. 3, 4 correspond to the wide ($d = 50nm$) rectangular barrier ($V_0 = 0.00025eV$) and well ($V_0 = -0.00025eV$), respectively. In all cases $m = 0.067m_e$ where m_e is the mass of an electron.

So, in the limit of weak scattering potentials $d_{eff}/d = 1$ for all values of k. That is, our definition of the tunneling time guarantees the passage to the free particle case, in this limit. However, this is not the case for the phase and dwell times. By these concepts, in the limit of weak scattering potentials the time spent by a slow particle in the barrier region differ essentially from $\tau_{free}(k; d)$. In particular, in comparison with τ_{free} the dwell-time concept predicts anomalously short times spent by a slow particle in the barrier region.

To explain this fact, let us remember once more that the dwell time was introduced as the ratio of the probability to find a particle in the barrier region to the incident probability flux. It is clear that for $\kappa_0 \neq 0$, in the long-wave limit, the number of particles in the barrier region is proportional to k^2. That is, a particle with a however small value of k does not enter the barrier. Taking also into account the fact that the incident flux $\sim k$ in this limit, we obtain $\tau_{dwell} \sim k$ instead of $\tau_{dwell} \sim k^{-1}$.

This property of the dwell time is kept for a however small value of $\kappa_0 d$. That is, strictly speaking, the dwell-time concept does not imply the passage to the case of a free particle, in the limit of weak scattering potentials. This fact evidences that the dwell time is ill-defined and cannot serve as the characteristic time to describe the dynamical properties of a tunneling particle.

As is seen from the figures, the phase-time concept does not guarantee the above passage, too. For slow particles, $|D_{phase}|/d$ may be however large, being negative by value in the case of wells.

Note, to compare the properties of the phase times and ours is of a particular importance. "Where do transmitted particles start from, on the average?" is the main intriguing question to arise in this case. Remind, in contrast with the phase-time concept to imply that in the above setting the tunneling problem transmitted particles start, on the average, from the point $x = 0$, our formalism says that this point is x_{start}.

Exps. (2) and (4) show that in the limit of weak scattering potentials the phase-time concept predicts an abnormal divergence of the transmission time at $k \to 0$. We have to stress that such a behaviour of the phase time takes place even if the transmission coefficient, T, approaches unit. Indeed, let us consider the case when

$\kappa_0 \neq 0$ and $k = d/\lambda^2 \to 0$; λ is fixed. One can easily show that in this case

$$T = \frac{4}{4 + \lambda^4 \kappa_0^4},$$

that is, T is constant in this limit. For $V_0 > 0$ we have

$$\frac{D_{dwell}}{d} = \frac{4}{4 + \lambda^4 \kappa_0^4}, \qquad \frac{D_{phase}}{d} = \frac{2\lambda^4 \kappa_0^2}{4 + \lambda^4 \kappa_0^4} \cdot \frac{1}{d^2};$$

for $V_0 < 0$,

$$\frac{D_{dwell}}{d} = 0, \qquad \frac{D_{phase}}{d} = -\frac{2\lambda^4 \kappa_0^2}{4 + \lambda^4 \kappa_0^4} \cdot \frac{1}{d^2}.$$

In both the cases

$$\frac{d_{eff}}{d} = 1, \qquad x_{start} = -D_{phase}.$$

As is seen, if $\lambda^4 \kappa_0^4 \ll 4$, the barrier is transparent in this limit. What is more important, for wells $D_{dwell} = 0$ even when $T \approx 1$. As regards D_{phase}, it diverges in this limit, both for barriers and wells. This means, in turn, that in this case x_{start} in our approach diverges too (of course, for finite wave packets the average value of $|x_{start}|$ does not exceed the wave-packet's half-width).

Figs. 5-8 show the time evolution of the wave packets Ψ_{full}, Ψ_{tr} and Ψ_{ref} to describe all, transmitted and reflected particles, respectively. The potential barrier considered is a rectangular well: $V_0 = -712 eV$, $d = 1.08 \times 10^{-5} nm$; in all cases $m = 0.067 m_e$. At $t = 0$ (see Fig. 5) the state of a particle is described by the Gaussian wave packet whose half-width equals $15 nm$, the average particle's energy equals $0.00641 eV$; $a = 70 nm$. In this case the norm, $\overline{\overline{R}}$, of Ψ_{ref} equals $6.5 \cdot 10^{-3}$.

As is seen from Figs. 6-8, this weak scattering potential unexpectedly strongly changes the shape of the "full" wave packet. When the peak to correspond, at early times, to the CM of the incident wave packet arrives at the point lying at some distance of the barrier, a new peak to correspond to the CM of the transmitted wave packet simultaneously appears behind the barrier region (see Fig.7).

By the SWPA, the transmission time is negative in this case. This property of $\Psi_{full}(x)$ is usually interpreted as the evidence of an ultrafast or even superluminal propagation of a particle through the barrier. However, by our approach, such interpretation of the wave-packet tunneling is wrong. The above behaviour of the wave-packet's peaks is inherent to interference maxima. It is evident that the expectation value of the particle's position cannot behave in such a manner. In our approach, for any t, this value for transmitted particles should be calculated over the wave packet Ψ_{tr} whose motion near the barrier region is regular.

The above property of the "full" wave packet shows once more that there is no direct causal link between the transmitted and incident wave packets. By our approach,

the tunneling process is a combined stochastic process consisting from two alternative ones, transmission and reflection, provided that $\Psi_{full} = \Psi_{tr} + \Psi_{ref}$, where Ψ_{tr} is the wave function to describe transmission and Ψ_{ref} is that to describe reflection (see [7]).

It is important to remind here that the norms of Ψ_{full} and Ψ_{tr} are practically equal in the case investigated. However, due to the interference terms (which are proportional to $\sqrt{\overline{R}}$, rather than $\overline{R}$) the difference between the behaviour of the elementary state Ψ_{tr} and combined state Ψ_{full} (see [7]) is essential. In particular, unlike the "full" wave packet, the shape of $|\Psi_{tr}(x)|^2$ remains practically unaltered during the scattering event (see Figs. 6-8). The distance ($\approx 2 nm$) between the average starting points of all and transmitted particles is well larger than the width of this transparent barrier.

Remind (see [7]), the incoming wave to describe transmitting particles with a given value of k differs from that for all particles by the factor $\exp(\pm i\gamma(k))$ where $\gamma(k) = \arctan\sqrt{R/T}$; $R = 1 - T$. Since $|x_{start}| = |R'/[2\sqrt{RT}]|$ this shift of the average starting point may be very large even if the reflection coefficient $R(k)$ is small by value; for the key role here is played by $R'(k)$; the prime denotes the derivative on k. A similar situation arises in the case of a resonant tunneling.

V. RESONANT TUNNELING

One can easily show that the dwell and phase times, distinguished drastically in the long-wave limit, coincide with each other in the case of a resonant tunneling. Namely, for $E > V_0$, for $\kappa d = n\pi$ ($n = 1, 2, \ldots$), where $k = \sqrt{\beta\kappa_0^2 + n^2\pi^2/d^2}$, we have

$$\frac{D_{phase}}{d} = \frac{D_{dwell}}{d} = \frac{\kappa^2 + k^2}{2\kappa^2} = 1 + \frac{\beta\kappa_0^2 d^2}{2n^2\pi^2}.$$

At the same time, if n is even, then

$$\frac{d_{eff}}{d} = \frac{k^2}{\kappa^2} = 1 + \beta\frac{\kappa_0^2 d^2}{n^2\pi^2};$$

if n is odd,

$$\frac{d_{eff}}{d} = \frac{k^2 - \beta\kappa_0^2}{\kappa^2} \equiv 1;$$

besides,

$$\frac{x_{start}}{d} = (-1)^n \frac{\beta\kappa_0^2 d^2}{2n^2\pi^2}.$$

Note, near the resonance point k_r the transmission coefficient can be written as $T(k) = [1 + a_0^2(k - k_r)^2]^{-1}$, where a_0 is a length to characterize the resonance. It is evident that $|x_{start}| = a_0$. So that, the narrower the resonance peak on $T(k)$, the larger is the value of $|x_{start}|$.

VI. ABOUT THE POSSIBILITY OF MEASURING THE AVERAGE STARTING POINT OF TRANSMITTED PARTICLES

Of course, a new and important result to arise in the framework of the separate description of transmission and reflection is that the average starting point of transmitted (and reflected) particles does not coincide with the initial position of the CM of the incident wave packet to describe the whole ensemble of particles. From the theoretical point of view, we deal here with three wave fields, Ψ_{full}, Ψ_{tr} and Ψ_{ref}, where $\Psi_{full} = \Psi_{tr} + \Psi_{ref}$. Thus, it is not surprising that due to the interference between Ψ_{tr} and Ψ_{ref} the starting points of the CMs of the last two wave packets do not coincide, in the general case, with that of the incident wave packet.

This means, in particular, that the average starting point of transmitted (or reflected) particles and that of all particles cannot be measured simultaneously. Let O_{inc} be an observer to study particles before the scattering event. Besides, let O_{tr} and O_{ref} be observers to study transmitted and reflected particles, respectively. It is evident that from the viewpoint of O_{inc} particles start, on the average, from the point $x = 0$. This observer cannot separate transmission and reflection. On the contrary, O_{tr} (O_{ref}) cannot measure the average starting point for all particles, but he can measure the average starting point of transmitted (reflected) particles. As is follows from our approach, he must find that particles transmitted by the barrier start, on the average, from the point x_{start}.

Of course, in verifying our formalism, measuring the value of x_{start} is of great importance. As in [8], we will exploit for this purpose the Larmor precession of a spin-1/2 particle in a small magnetic field. However, our aim is directly opposite, because timing the scattering particle have already been performed in our approach.

Let an infinitesimal constant uniform magnetic field **B** be applied along the z-axis, everywhere outside the interval $[a - l, b + l]$ on the x-axis; we assume that $a - l > 0$, moreover $a - l \gg l_0$ and $l \gg l_0$; l_0 is the half-width of the incident wave packet. When a spin-1/2 particle moves in this region, the axis of its (average) spin will rotate around the z-axis with a constant frequency $\omega_L = g\mu B/\hbar$, where g is the gyromagnetic ratio, μ is the absolute value of the magnetic moment of the par-

ticle. Of course, we assume that at $t = 0$ the spin axis is not parallel to the z-axis. Moreover, we assume also that at this moment the expectation values $< S_x >$ and $< S_y >$ of the x- and y-components of the spin are equal: $< S_x^{(0)} >$ and $< S_y^{(0)} >$.

Let the spin of a particle with the well-defined value of k (the corresponding wave packet is narrow in k-space) be detected at that instant of time, t_{det}, when the CM of the transmitted wave packet arrives at the point $x = b + L$; $L - l \gg l_0$. Thus, on the average, the particle moves under the magnetic field during the time $\Delta t_{in} + \Delta t_{out}$, where $\Delta t_{in} = \frac{m}{\hbar k}(a - l - x_{start})$, $\Delta t_{out} = \frac{m}{\hbar k}(L - l)$. Hence, the expectation values $< S_x >$ and $< S_y >$ for transmitted particles, at the instant of time $t = t_{det}$, read as

$$< S_x >= \frac{< S_x^{(0)} >}{\sqrt{2}} \cos \left[\frac{m\omega_L}{\hbar k}(a + L - 2l - x_{start}) + \frac{\pi}{4} \right]$$

$$< S_y >= \frac{< S_y^{(0)} >}{\sqrt{2}} \sin \left[\frac{m\omega_L}{\hbar k}(a + L - 2l - x_{start}) + \frac{\pi}{4} \right]$$

From this it follows that for this moment in the case of equal values of $< S_x^{(0)} >$ and $< S_y^{(0)} >$ we have

$$x_{start} = a + L - 2l + \frac{\hbar k}{m\omega_L} \left[\frac{\pi}{4} - \arctan \left(\frac{< S_y >}{< S_x >} \right) \right] (6)$$

Note, according to the SWPA the expression in the right-hand side of (6) should be equal to zero.

Thus, Exp. (6) can serve as the basis for the experimental checking of our approach. They provide the way of calculating the value of x_{start} from experimental data on measuring the particle's spin. We have to stress once more that these expressions describe the case when an external magnetic field is applied to the spatial regions where the incident and transmitted wave packets evolve freely.

Note, Exp. (6) is valid also for finite wave packets. However, now the expectation values of k for the incident ($< k >_{inc}$), transmitted ($< k >_{tr}$) and reflected ($< k >_{ref}$) wave packets are different. Thus, k in (6) should be replaced in this case with $< k >_{tr}$.

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Figure captions

Fig. 1. D_{eff}/d versus E/V_0 for the narrow ($d = 0.5nm$) rectangular barrier ($V_0 = 0.25eV$).

Fig. 2. D_{eff}/d versus E/V_0 for the narrow ($d = 0.5nm$) rectangular well ($V_0 = -0.25eV$).

Fig. 3. D_{eff}/d versus E/V_0 for the wide ($d = 50nm$) rectangular barrier ($V_0 = 0.00025eV$).

Fig. 4. D_{eff}/d versus E/V_0 for the wide ($d = 50nm$) rectangular well ($V_0 = -0.00025eV$).

Fig. 5. The x-dependence of $|\Psi_{full}|^2$ (solid line) which represents the Gaussian wave packet with $l_0 = 15nm$ and the average kinetic particle's energy $0.00641eV$, as well as $|\Psi_{tr}|^2$ (open circles) and $|\Psi_{ref}|^2$ (dashed line) for the rectangular well ($V_0 = -712eV$, $d = 1.08 \times 10^{-5}nm$, $a = 70nm$); $t = 0$.

Fig. 6. The same as in Fig. 5, but $t = 29ps$.

Fig. 7. The same as in Fig. 5, but $t = 33.5ps$.

Fig. 8. The same as in Fig. 5, but $t = 38ps$.

E/V$_0$